

# THE INFLUENCE OF AXIAL COMPRESSION ON THE ELONGATION OF PLASTIC HINGES IN REINFORCED CONCRETE BEAMS

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## SUMMARY

Four reinforced concrete beams were tested to investigate the effect of axial force on the elongation that occurs in plastic hinge zones. The beams, designed to replicate the corresponding detail from a multi-storey moment resisting frame, were cyclically tested whilst being subjected to different constant axial forces. The axial forces, up to  $0.0833 A_g f_c'$ , were applied to replicate the beam-slab interaction that acts to restrict elongation. The influence of moment and shear on elongation was tested respectively by using different reinforcement and length details for different tests. All the beams tested were 450 mm deep and 200 mm wide. A detailed account of beam construction, test setup, testing procedure and test results are provided in this paper.

## INTRODUCTION

The seismic design of ductile reinforced concrete frames in New Zealand requires a capacity design approach with hinging in the beams, ensuring a weak-beam strong-column design. This desired failure mode is achieved by proportioning and detailing the members so that flexural yielding is restricted to selected plastic hinge regions, in the beams at column faces and at the base of columns. At other regions where inelastic deformations are not desirable, the members are designed so that their strength exceeds the demands imposed from the overstrength actions sustained in the plastic hinges. The requirements for this design approach are governed by the New Zealand Standard for the Design of Concrete Structures, NZS 3101:1995 [1].

What is not emphasised in NZS 3101 [1] is that the mechanism of beam hinging requires the occurrence of elongation in the hinge zone [2]. Restraint against elongation as a consequence of the stiffness and strength of the diaphragm can result in a large compression force in the beam, significantly increasing the beam's strength. Tests by McBride [3], Lau [4] and Matthews [5] have demonstrated this phenomenon. In the event of an earthquake this strength enhancement is likely to give rise to unpredicted structural behaviour. This unpredictable behaviour could lead to floors within multi-storey buildings collapsing, soft stories, or in a worst case scenario total building collapse.

Tests conducted at the University of Auckland have observed the phenomenon of elongation over the past 25 years. This was first reported on by Fenwick and Fong [6] in 1979 who cyclically tested 10 reinforced concrete beams. Two of these tests had unstressed pre-stressing strands that were de-

bonded from the concrete, one laid parallel to the reinforcement the other at an angle of 20 degrees. A significant reduction in elongation was measured in the beam containing the parallel pre-stressing strands. Elongation of 13.7 mm, equating to 2.7% of the total beam depth, was recorded compared to 19 mm, 3.8% of the total beam depth, for the same beam when tested with no pre-stressing strands. No appreciable difference in elongation was seen when the pre-stressing strands were laid at an angle of 20 degrees.

In 1984 four reinforced concrete beams subjected to different axial forces were tested by Spencer. These tests were not reported on until 1998 by Issa [7] who performed two additional tests; one subjected to an axial compression force the other an axial tension force. Elongation was found to reduce to 1.3% of the total beam depth when subjected to the maximum compressive force, compared to 3.5% for an equivalent beam with no axial compression force. Elongation further increased to 4.3% of total beam depth when subjected to the axial tensile force.

In 1999 Matti [8] conducted two further tests involving beams subjected to axial force. The first test showed that the shear component of displacement, 29% of the total end displacement, was greatly reduced with a compressive axial force and overstrength values recorded at higher ductility load cycles. Rapid strength degradation at higher ductility cycles and higher shear displacements were found to occur with an axial tensile force. This first beam test of Matti showed that beams in moment-resisting reinforced concrete frames have the potential to reach higher strengths than calculated with code design rules due to the restrictive nature of beam-slab detailing.

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The methods used in the design of structural systems rely heavily on the results of computer analyses. An example of this reliance is the now well accepted “dynamic magnification factor” that is used to amplify the column actions obtained from a linear elastic analysis. The dynamic magnification concept was developed twenty years ago [9] from the results of what could be considered today as relatively simple inelastic time history analyses. These time history analyses did not include the influence of elongation. Recent work by Lau [4] has indicated the need for more data to allow researchers to develop a more accurate beam hinge model. Numerical models of structures using such a hinge would automatically include the appropriate strength enhancement of the beam arising from the attempts of the hinge to elongate. These more accurate models would provide researchers (and Code developers) with better guidance on suitable column strength requirements in the seismic design of reinforced concrete frames.

It is worthwhile noting that the design rules in NZS 3101 only provide guidance to beam strength enhancement resulting from the interaction of a standard floor slab. Little guidance is provided for estimating the increase in stiffness of beams and the consequences of using a precast prestressed floor system. In addition, the Loadings Standard [2] allows designers to use more sophisticated nonlinear time history and pushover analyses that do not incorporate the phenomenon of elongation to show that structures that do not comply exactly to NZS 3101 are adequate. As a consequence, these sophisticated analyses are most likely underestimating the strength of the beams, and consequently underestimating the required strengths of columns in seismic frames.

The objective of the tests described in this paper was to provide valuable data and insight into the behaviour of beam hinges thus enabling the development of a workable numerical model for an elongating plastic hinge.

## DESIGN OF TEST SPECIMENS

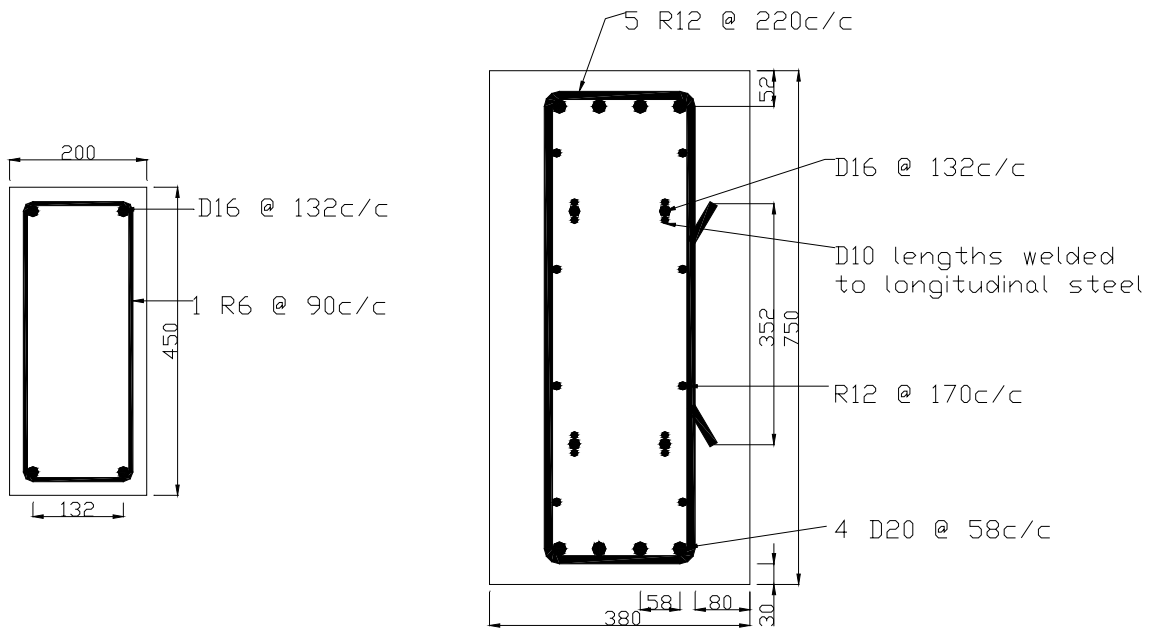
To examine the influence of an axial compression force on the stiffness, strength and elongation characteristics of a reinforced concrete beam under cyclic loading, the test unit shown in Figures 1 and 2 was conceived. The test unit comprised of two beams cantilevering from a heavily reinforced thickened central block. The central block represented the column joint zone and provided a

means to allow the two beams to be fastened at a suitable height for testing. The test units were designed so that in addition to axial force, the influence of the percentage of reinforcement (moment) and shear level, on the performance of the beams could be assessed.

Each cantilever beam in a test unit was designed to represent half a beam within a multi-storey moment resisting reinforced concrete frame. The design of the beams was based on the Cement and Concrete Association of New Zealand’s “Examples of Concrete Structural Design to New Zealand Standard 3101:1995” more commonly known as the “Red Book” [10]. The cross-section dimensions of the beams were a half scale of the beams used for the 10 storey office building in the “Red Book”, resulting in dimensions of 450 mm deep and 200 mm wide.

A summary of the beams described in this paper is provided in Table 1. These beams form a subset of a total of twelve beams that will be tested in this research programme. The moment capacity of the test specimens will be varied by using two different percentages of steel reinforcement; one close to minimum steel, the other heavily reinforced. The weaker beams consist of 2-D16 bars top and bottom ( $\rho=0.00483$ ) whilst the stronger beams consist of 6-D16 bars top and bottom ( $\rho=0.01525$ ). A typical reinforcement layout can be seen in Figure 1. To prevent yielding of the longitudinal bars from the beam extending into the thickened central region, 1-D10 bar was welded top and bottom to the longitudinal bars in the central block. The transverse reinforcement was designed to the requirements of NZS 3101, clause 8.5.4.3, assuming that the potential plastic hinge zone (PPHZ) was located at the face of the thickened central region and extending along the beam a distance of  $2D$  or 900 mm. 1-R6 full depth stirrup @ 90 mm c/c spacing was used inside the PPHZ with the spacing governed by the smaller of  $d/4$  or  $6d_b$ , where  $d$  is the distance from the extreme compression fibre to the centre of the tension reinforcement and  $d_b$  is the diameter of the stirrups. Outside the PPHZ 1-R6 full depth stirrup @ 200 mm c/c spacing was used, governed by spacing  $\leq d/2$ . 30 MPa concrete was used for all beams and all reinforcing steel was Grade 300.

Samples of steel were tension tested and found to have an average yield stress of 319 MPa. Concrete cylinders were taken during construction, then crushed on the day of testing and found to have an average yield stress of 34 MPa.



(a) Beam Reinforcement

(b) Reinforcement of Thickened Block

Figure 1 Reinforcement Layouts

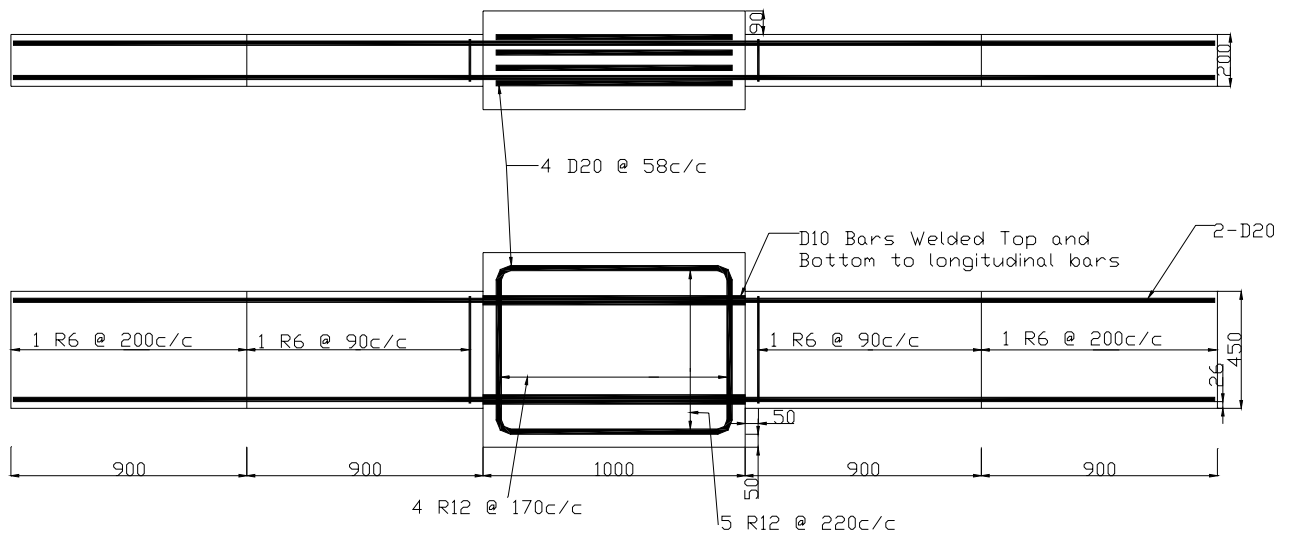


Figure 2 Test Unit Dimensions

The lengths of the beams were chosen to obtain different shear levels for a specific flexural strength. Typically design length/depth ratios in seismic frames vary between  $4 < L/D < 8$  where L is the face to face length of the beam between two columns and D is the total beam depth. For the test

specimens, ratios of 4.5 and 7.5 were chosen giving beam lengths of 1.1 m and 1.6 m respectively. These beam lengths represent half of the total bay, the distance from the face of a column to the centre of the bay, the point of vertical load application for testing.

Unit	Longitudinal Reinforcement (Top & bottom)	Length (m)	Axial Force	
			kN	$N^*/A_g f'_c$
1	2-D16	1.6	0	0
2	2-D16	1.6	127	0.0417
3	2-D16	1.1	0	0
4	2-D16	1.1	255	0.0833

**Table 1 Summary of Unit Dimensions**

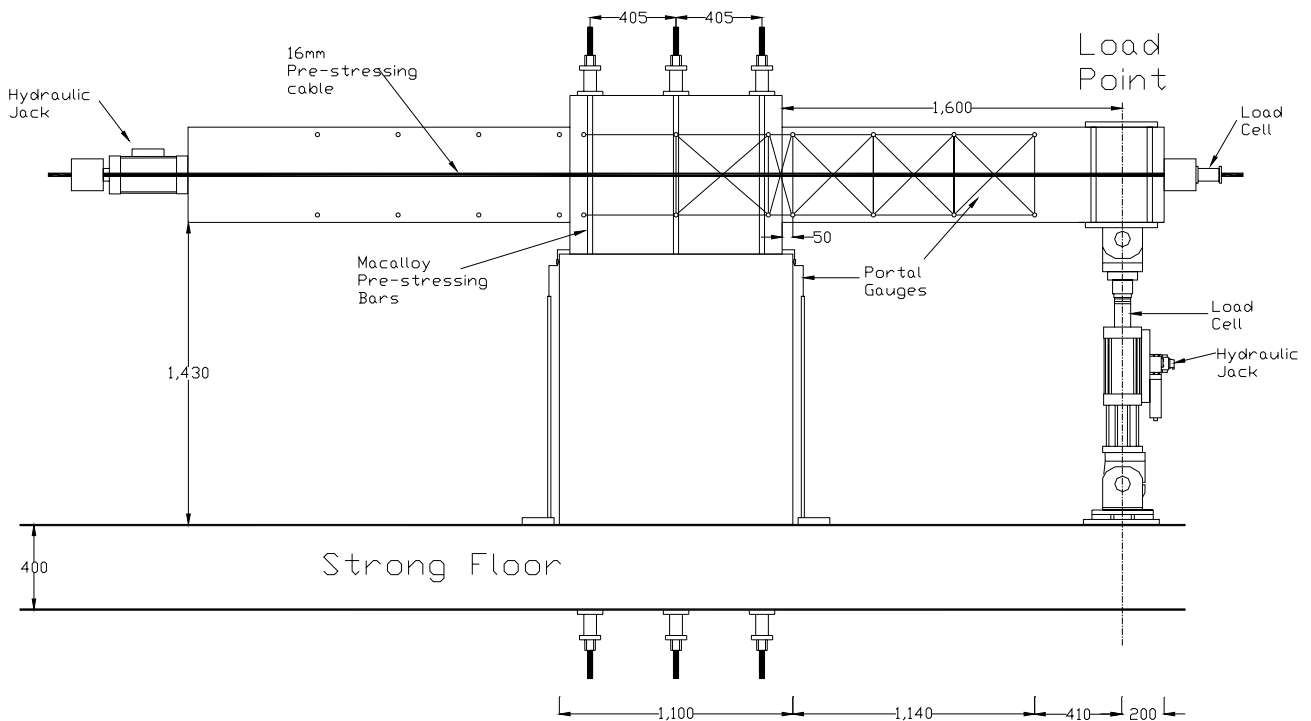
**TEST SETUP**

Figure 3 shows the test setup. Each test unit was fastened securely down to a large, heavily reinforced, concrete pedestal. The pedestal was specifically cast for the project and utilized steel angles along its top and bottom edges to avoid crushing of the concrete in these areas during the cyclic testing. Six plastic ducts were cast through the pedestal, at 405 mm centres, allowing the pre-stressing bars that were used to stress the test units to the floor to pass through.

The test units were placed on top of the pedestal with a layer of mortar between them and the

pedestal, to provide an even surface. A thick steel plate was then placed on top of the thickened region of the unit, using mortar to create an even placement surface. Six Macalloy pre-stressing bars were run through holes in the plate, around the unit, then through the pedestal and strong floor and stressed down. The beam was allowed to displace vertically in the same direction as the jack but restrained from moving laterally and torsionally at its free end. The restraint was achieved by the attachment of two parallel rods bolted to the top and bottom sections of the beam and tied back to a reaction frame. The torsional restraint system can be seen in Figure 4.

To provide axial compression to the beam, two 16 mm diameter steel cables were run along the sides of the test unit at mid height. At the ends of both beams the cables passed through an RHS section and were secured. A 30 tonne jack was placed between the RHS and the beam at the end not being tested. At the other end, a 25 tonne load cell was placed on each cable and separated from the end of the beam by another RHS section. The sum of the forces in the load cells was the total axial compression force applied to the beam. A 50 tonne reversing jack was used to apply the vertical cyclic load.



**Figure 3 Test Setup**



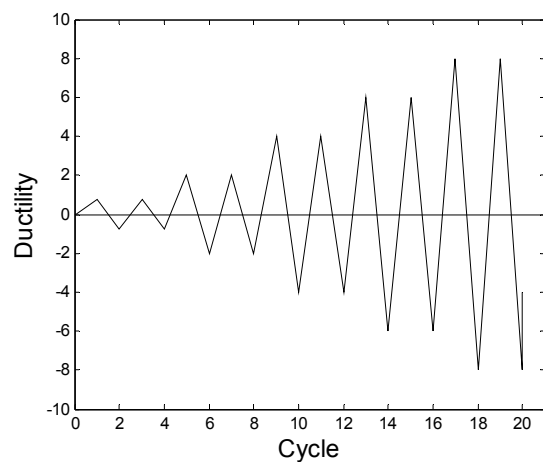
**Figure 4 Torsional Restraint System and Jack Setup for Unit 1**

Each unit was instrumented extensively with portal displacement gauges as shown in Figure 3. These measured the change in length between each measurement point and allowed the contribution of flexure, shear and elongation over each section of the beam to the final end displacement to be calculated. In addition, the total displacement of the point of load application was measured using a large portal displacement gauge and a turnpot in parallel. The portal gauge was more accurate than the turnpot but had the displacement restriction of  $\pm 60$  mm. The turnpot, while less accurate, was used for larger displacements at the large ductility cycles. Other portal gauges were used to measure displacements between the unit and the pedestal and between the pedestal and the strong floor. These gauges allowed the component of displacement due to rigid body motion to be determined.

#### **TEST REGIME**

The frequently used load history reported by Park [11] was used for all tests. This required the unit to be subjected to two complete loading cycles at specified ductility amplitudes, and is shown in Figure 5. One complete cycle comprised both a positive and a negative amplitude. Initially the loading was force controlled and the specimen was loaded to 0.75 times the nominal design strength,

for two complete cycles. Using linear interpolation of the recorded displacements for these elastic cycles, the ductility 1 displacement was determined. The subsequent loading cycles then became displacement controlled based on this measured ductility 1 value. The loading procedure was repeated until failure occurred. Failure was defined as a reduction in member strength of 20% of the maximum measured flexural strength for that direction of loading.



**Figure 5 Loading Cycle**

The nominal beam design strengths, for the two units with an axial force, were adjusted to allow for the strength increase due to the compressive force.

## TEST RESULTS

Figure 6 illustrates the force-displacement curves for the four units tested. From these figures the increase in beam strength with the application of axial compression force can be seen. Figure 7 illustrates the elongation of the units with increasing vertical displacement. Figure 7(a) and 7(c) depicts elongation in the absence of axial force and these compare well with previous research, Liddell [12].

### Unit 1

Unit 1 was 1.6 m long with no axial force present. Its measured yield strength was 56.8 kNm and compared well with the design moment using tested material properties of 52.0 kNm.

Unit 1 experienced a maximum elongation of 10.2 mm or 2.3% of total beam depth. An increase in elongation during the test was appreciable up to the end of the second ductility 8 cycle, where the rate of elongation slowed due to buckling of longitudinal reinforcement. With no axial force, the increase in elongation occurred when the displacement of the beam was increased to a greater ductility level. It then remained approximately constant for the remainder of the beam displacement at that ductility level until the zero displacement point where it would increase again. The elongation history is shown in Figure 7(a).

### Unit 2

With the application of an axial force of  $0.0417 A_g f_c'$  the yield strength increased to 85.6 kNm. The measured strength was a little above the calculated nominal yield strength of the beam (including the strength enhancement from the axial force) of 80.3 kNm.

The maximum elongation calculated was 7.9 mm or 1.8% of total beam depth with the axial force present. The development of elongation in this test differed markedly from the process observed for unit 1 where no axial force was applied. In this test,

Figure 7(b), as with unit 1 the elongation increased largely on the application of the first part of the displacement cycle to a greater ductility, but on reversing the displacement, the elongation reduced. Typically the elongation reduced to a minimum when the beam was at zero displacement. It should be noted that this was not the position of an “unloaded” beam, can be seen in Figure 6(b) that at zero displacement, the moment in the beam was approximately 40% of the maximum.

### Unit 3

Unit 3 was 1.1 m long and tested without axial force. The beam yield strength of 52.0 kNm compared well to the calculated yield strength of 52.0 kNm using tested material properties.

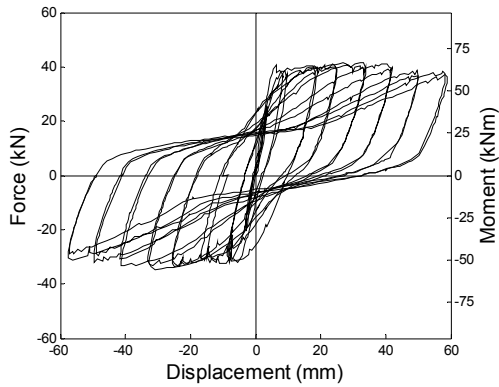
Figure 7(c) shows that unit 3 experienced a maximum elongation of approximately 11.2 mm or 2.5% of the total beam depth. The response of this unit was similar to that of unit 1. Again elongation was appreciable until the onset of buckling of the longitudinal reinforcement around the ductility 8 cycles.

### Unit 4

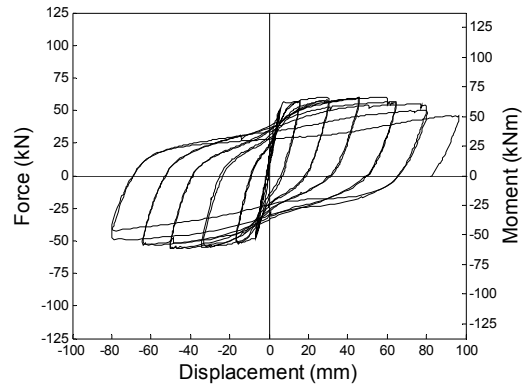
Unit 4 was 1.1 m long and was subjected to an axial force of  $0.0833 A_g f_c'$  during testing. A yield strength of 107.8 kNm was recorded during testing and this compared well to the calculated design strength of 106.3 kNm.

The elongation response of this unit, shown in Figure 7(d), was similar to that of unit 2. As unit 4 had a large axial compressive force, very little elongation occurred except the amount required as a result of the geometric necessity of rotating the beam hinge. The maximum recorded elongation was 4.3 mm or 0.96% of the total beam depth.

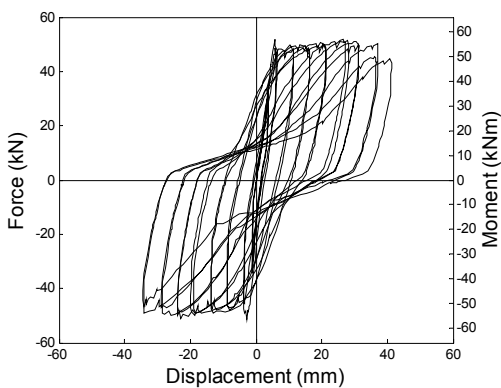
Figure 7(d) shows the increase in elongation that occurred from the removal of the axial compression force at the conclusion of testing. The beam was then displaced a further two loading cycles. The beam was still able to hold its initial design moment when subjected to a complete ductility 14 cycle. Rapid elongation was seen at these later stages to levels similar to those of unit 3 where no axial force was present.



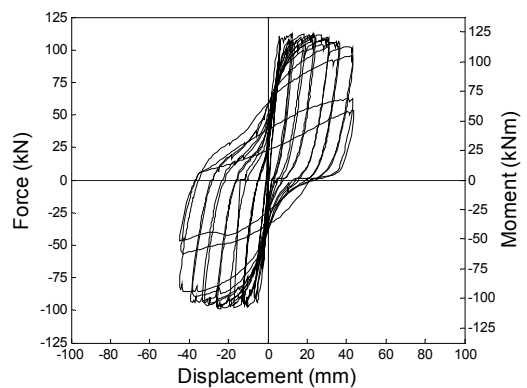
(a) Unit 1 (Long, axial force =  $0 A_g f_c'$ )



(b) Unit 2 (Long, axial force =  $0.0417 A_g f_c'$ )



(c) Unit 3 (Short, axial force =  $0 A_g f_c'$ )



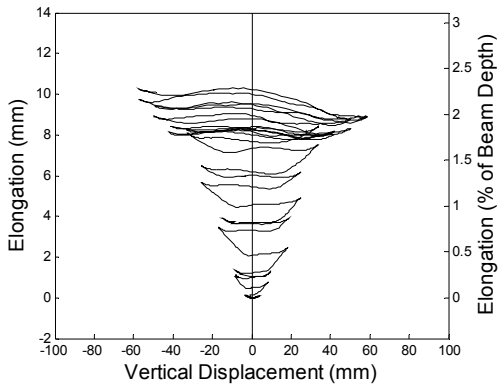
(d) Unit 4 (Short, axial force =  $0.0833 A_g f_c'$ )

**Figure 6 Force versus Displacement Curves**

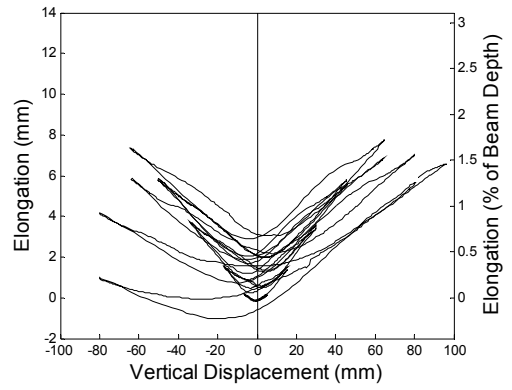
Examination of Figures 7(b) and 7(d), that depict elongation in the presence of axial force, reveals a V-shaped plot. These figures illustrate a reduction in elongation from the position of maximum beam displacement occurring in a cycle to the point of zero displacement. The reduction is particularly noticeable for unit 4 where the axial force was significant enough to completely suppress any elongation at zero beam displacement throughout testing. Figure 8 compares the elongation of the four units as measured at the peak negative ductility (beam displacement). The figure shows an appreciable decrease in elongation with the presence of axial force for units 3 and 4. Units 1 and 2 show no appreciable difference in elongation at equivalent ductility cycles until the 8ii cycle, from a comparison of Figures 7(a) and 7(b) this can be

seen to be false everywhere except at these maximum values.

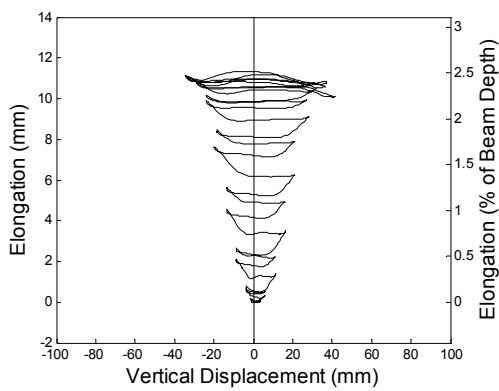
The above finding is significant when making a comparison of test results. Past practice when comparing elongation has been to compare results at levels of maximum force. This has little influence on the magnitude of the number plotted as there is negligible difference in the level of elongation between the values at maximum displacement through to zero displacement. In the presence of axial force, appreciable difference can be noted between the elongation at maximum and zero displacement, indicative of the V-shaped nature of the plot. This would significantly alter the appearance of Figure 8 if elongations at zero displacements were plotted.



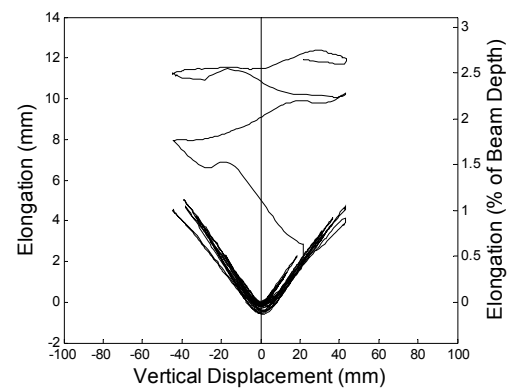
(a) Unit 1



(b) Unit 2



(c) Unit 3



(d) Unit 4

Figure 7 Elongation versus Vertical Beam Displacement

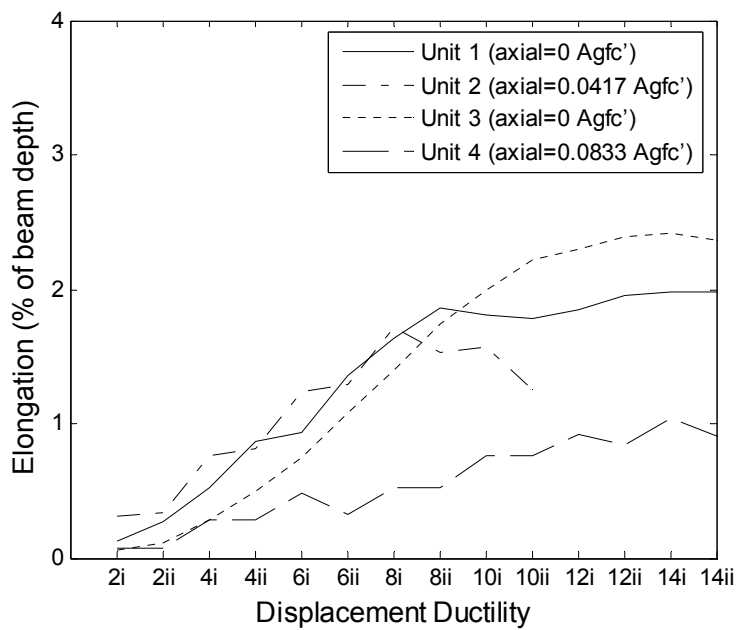
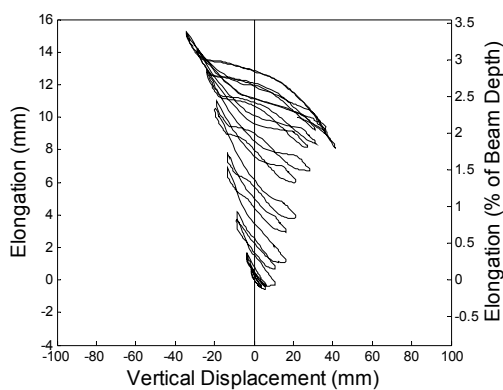
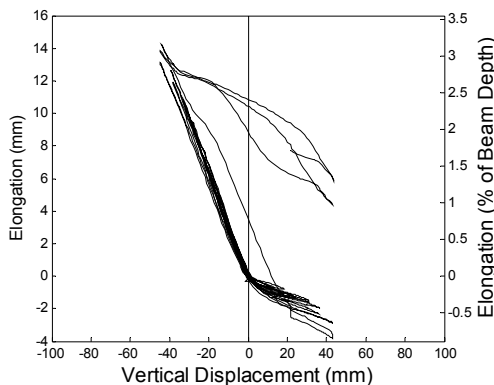


Figure 8 Elongation Growth with increasing Ductility

Figure 9 illustrates the elongation of the top reinforcing bars for units 3 and 4. From a comparison of these figures it can be seen that the top reinforcement experienced similar magnitudes of maximum elongation of approximately 2% for ductility 8 cycles with and without axial force present. With axial force present this elongation was not retained and upon returning to a point of zero beam displacement is fully suppressed. This was significant as the position of the top reinforcing bars is generally the level at which floor-slabs are connected to supporting beams. To prevent strength increase within these supporting beams this elongation needs to be allowed to fully develop.



(a) Unit 3



(b) Unit 4

Figure 9 Elongation of Top Reinforcement

### Conclusions

Through testing, without axial restraint, beams were found to elongate approximately 2% of total beam depth when cycled to approximately ductility

8 deformation. These elongations were consistent with previous test results.

The influence of axial compression force on a beam was to reduce the maximum mean elongation, as well as significantly enhance the strength of the beam. For an axial compressive force of  $0.0833 A_g f_c'$ , elongation was found to reduce to 1.0% of total beam depth at levels of maximum beam displacement. However the elongation was completely suppressed at zero beam displacement.

For a smaller axial compressive force of  $0.0417 A_g f_c'$  elongation was found to reduce to 1.8% of total beam depth at levels of maximum vertical displacement. The elongation however was suppressed to approximately 0.5% at a point of zero vertical displacement.

The elongation of top reinforcing bars in beams cyclically tested, when subjected to an axial compressive force, was found to reach levels similar to beams tested without axial force. The elongation was to allow for the geometric necessity of rotating the beam hinge and was suppressed upon unloading to a level of zero beam displacement. To prevent strength increase in beams this elongation needs to be allowed to fully develop.

Further tests need to be performed to further assess the influence of axial compressive forces on the elongation and strength enhancement of reinforced concrete beams.

### ACKNOWLEDGEMENTS

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